Experimentally validated stochastic geometry description for textile composite reinforcements

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Abstract

The uncertain quality of composites, due to variability in the mechanical response, forces design engineers to employ high safety margins to ensure that the design requirements are met. Especially for textile composites, an improved assessment of the quality of any composite material is achieved by identification and simulation of the inherent uncertainty in the reinforcement geometry. This paper presents such a comprehensive multi-scale strategy to develop realistic stochastic replicas of a composite material, with emphasis on the identification step. First, the scatter in the tow reinforcement is characterised on the short-range (meso-scale) and long-range (macro-scale) from high-resolution images. Next, a probabilistic uncertainty quantification method is proposed to analyse the variability of each path parameter in terms of average trend, standard deviation and correlation information. This set of statistical information is essential to reproduce the random textile geometry in a numerical simulation approach. The multi-scale framework delivers representative models in the WiseTex format and is demonstrated for a carbon-epoxy 2/2 twill woven composite produced by resin transfer moulding.

Keywords: A. Textile composites, C. Computational Mechanics, C. Multiscale modelling, C. Probabilistic methods, C. Statistics

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1. Introduction

The advantages of using composites in structural applications are well known. Though, the introduction of composites is hampered by the relatively high cost of raw material and the large scatter of performance characteristics. In order to assure the design requirements, high safety factors and strict manufacturing tolerances are enforced. An improved assessment of the quality of any composite part is achieved by identifying the irregularity in the tow reinforcement, which forms a direct link with the variability in the macroscopic performance. The effect of geometrical imperfections on the magnitude of variability in performance has already been reported with contributions considering elastic mechanical properties [1, 2], formability [3] and permeability [4, 5]. However, a complete statistical characterisation of the geometrical variation is still missing for many types of textile composites, while modelling approaches of textile geometry frequently omit or only partially introduce variation in the reinforcement path [6, 7, 8].

Specifically for textile composites, variation in the reinforcement structure is characterised at multiple scales. The micro-scale represents local features with reference lengths of 10^{-6} - 10^{-4}m. This is the scale of individual fibres that are bundled within a single tow. For larger reference lengths of 10^{-4} - 10^{-2}m, determined as the meso-scale, individual fibres cannot be distinguished anymore but appear in groups that correspond to the tows. The macro-scale averages out the small effects of the heterogeneous structure at tow level and permits to quantify any long-range variations with lengths of 10^{-2} - 10^{-0}m. Most types of composites are built using an ideal, periodic unit cell that can be identified at the meso-scale based on deterministic inputs. Such numerical models are an idealised representation of the textile composite since real physical samples consist of spatially distributed unit cells that differ from neighbour to neighbour, affecting the mechanical response.

A realistic modelling approach requires the introduction of scatter at the different levels and calibration with experimental work. This paper describes such a generic modelling approach for woven textile composites that can consist of multiple unit cells. The examined procedures are also applicable to characterise and simulate any other textile structure considering only minor adjustments. In general, the most suitable approach to obtain realistic representations of any composite material is by (i) collecting experimental data on the spatially correlated random geometrical fluctuations on the short-range, i.e. within the unit cell size, and long-range, and (ii) employing simulation techniques to represent the spatial variability using this statistical data as input information. In a next step, the variability in the macroscopic material properties can be predicted from these stochastic reinforcement descriptions.

The aim of the proposed multi-scale procedure is to reproduce the full internal structure of composite specimens that match the statistical information of a characterised composite sample in its final form. Virtual woven textiles spanning multiple unit cells are constructed using the concept of a periodic unit cell for generating the average reinforcement path over the extent of the entire composite. Next, zero-mean deviations are added to this average behaviour using techniques and results from preceded publications [9, 10, 11, 12]. The representative unit cell, unique and the same all over the composite volume, is replaced by a representation of spatially distributed unit cells where each unit cell is different.

2. Multi-scale framework

Realistic woven specimens are acquired that are replicas of the experimental samples following the multi-scale framework schematically presented in figure 1. The variability of each tow path is defined for the centroid coordinates (x, y, z), tow aspect ratio AR, tow area A and tow orientation in cross-section θ which fully describe a woven reinforcement; scatter in the matrix and fibre properties are not considered. Three successive steps can be distinguished to obtain such random representations:

1. Collection of experimental data and statistical analysis
2. Generating instantiations of stochastic textile reinforcement using multi-scale modelling
3. Construction of virtual specimens in a geometrical modelling software

This article describes the measurement and characterization of variability in the reinforcement structure of textile composites, described in section 3, and briefly discusses the use of such data to generate stochastic virtual specimens in sections 4 and 5. These latter steps require much more mathematical developments, which are of a completely different nature than step 1, and will be published in another publication which deals rather with mathematical modelling concepts. All steps are demonstrated on a carbon-epoxy 2/2 twill woven composite produced by Resin Transfer Moulding (RTM).

3. Collection of experimental data and statistical analysis (step 1)

3.1. Experimental framework

Different experimental techniques are selected to effectively investigate the geometrical variability present within the unit cell and propagating over several unit cells.

In a first step, it is recommended to acquire three-dimensional (3-D) images of the internal geometry of the sample via laboratory X-ray micro-computed tomography (micro-CT). This technique is efficient to quantify the short-range spatial information of the entire reinforcement in a single experiment. From the 3-D volume representation, equally spaced two-dimensional (2-D) slices are extracted in each weave direction to trace the tow path data along its length. Tows in cross-section are fitted with a geometrical shape using the freeware ImageJ from which the centroid coordinates and cross-sectional information can be deduced. Next, a statistical description of the tow parameters is obtained by applying the reference period collation method [13]. This approach groups tows that should be identical given the nominal periodicity of the textile, with such a representative tow named *genus*. Each characteristic is partitioned for each genus into periodic systematic variations and non-periodic stochastic deviations, over a grid with N_i locations chosen to be dense relative to the periodic lengths. The decomposition can be represented where ε represents one of the five parameters $\{\rho, z, AR, A, \theta\}$ and $\rho = y$ or x respectively for each genus:

$$\varepsilon_i^{(j,t,s)} =< \varepsilon_i^{(j,t,s)} > +\epsilon_i^{(j,t,s)} \quad (1)$$

with $\epsilon_i^{(j,t,s)}$ the zero-mean deviation from the systematic value $< \varepsilon_i^{(j,t,s)} >$ at location i ($i = 1..N_i$) along the tow j ($j = 1..N_j$) of tow genus t in ply s. The label s appears in laminates of stacked plies, but is unnecessary in single-ply samples. The systematic variation describes the average tow path parameter defined as the mean value of that parameter at each grid point along the reference period. Random deviations $\epsilon_i^{(j,t,s)}$ are further quantified in terms of standard deviation σ and correlation information. Correlation is computed using the Pearson's moment correlation coefficient for pairs of data with equidistant spacing ν taken at distinct locations on the same tow spaced by $k\nu$ (auto-correlation C_{auto}), and pairs of data taken at the same grid location but on other neighbouring tows, spaced by $k\nu'$ (cross-correlation C_{cross}). The direction of these dependencies is indicated in figure 2. In addition to the cross-dependencies between tows belonging to the same genus, correlation between two different genuses can also be verified, e.g. at the cross-over locations where different tow positions overlap. From the correlation data in terms of point spacings, a correlation length ξ is estimated which "measures the distance of two different stochastic field locations over which the correlation between the respective random variables approaches zero or a practically very small value" [8]. This particular length is determined by: (i) linear regression of the correlation data for the first few point spacings k in the case of limited data sets: $C(k) = 1 - k\delta/\xi$, with δ the equidistant point spacing in the correlation graph, or (ii) fitting the correlation trend by functions to at least half of the point spacings k for moderate or rich data sets: e.g. $C_{exp} = \exp\left(-\frac{\tau}{\xi}\right)$, with τ the absolute difference between two spatial coordinates.

Based on the derived auto- and cross-dependencies of the tow parameters, additional long-range characterisation could be advised. The magnitude of the computed correlation length of an individual parameter acts as indicator to evaluate if variation occurs within the unit cell size or exceeds the periodic length. This step should be supported by sufficient data from unit cell samples taken at different spatial locations and from distinct samples. Depending on the tow path parameter(s) that exhibit(s) such a long-range trend, additional micro-CT scans, optical imaging techniques or digital image correlation need to be performed. Spatial quantification of the out-of-plane centroid and cross-sectional variations always requires internal information, while the in-plane centroid can already be characterised using an optical scanning technique.

3.2. Application to a 2/2 twill woven textile composite

3.2.1. Material

As demonstration of the methodology, a 2/2 twill woven carbon fibre Hexcel fabric (G0986 injectex) [14] is considered. Each dry unit cell reinforcement consists of four equally spaced warp and weft tows with a nominal areal density of 285 g/m^2. The unit cell topology is given in figure 3 with the same periodic length for the warp (x-axis) and weft (y-axis) direction: $\lambda_x = \lambda_y$=11.43 mm. Laminated and single-ply carbon-epoxy samples are produced in a RTM process to quantify its geometrical variability on the short- and long-range. Two different *genuses* can be distinguished: warp tows are represented by one genus and weft tows as a second.

3.2.2. Short-range characterisation

Short-range variations are identified in [9] from a seven-ply sample using laboratory micro-CT. From the reconstructed 3-D volume with a voxel size of (6.75 μm)^3, 2-D slices are extracted in warp and weft direction. Although the image quality of the cross-sections is optimised using different filtering techniques, fully-automated material segmentation is not possible. The similar material density of the carbon fibres and epoxy resin results in a low contrast, as can be seen from figure 4. The required manual input for image segmentation limits the amount of slices to nineteen to analyse the structure of the sample in a reasonable time but capturing all the essential fluctuations in the tow path (set to every 0.75 mm along the tow path). The segmented tow shapes are fitted with an elliptical shape yielding information about the tow path centroids (ρ, z) with ρ=y for the warp tows and ρ=x for the weft tows, tow aspect ratio AR, tow area A and tow orientation θ in cross-section.

The tow path parameters of each genus, based on data from four tows ($N_j = 4$) for both the warp and weft genus, are described on an equidistant grid with sixteen points

($N_i = 16$) and a total grid length set to the unit cell period of the particular genus (λ_x or λ_y). These periodic lengths are derived from the experimental data using a minimisation algorithm described in [9]. In a next step, the genus of each tow parameter is partitioned as a non-stochastic, periodic systematic trend and non-periodic stochastic fluctuations using the reference period collation approach presented in equation 1 with $s = 1..7$. The presence of several plies maximises the data that can be derived from a single sample, but also permits to compare the statistics of each ply. Comparison of the ply systematic trends shows that no discernible differences are present for each single tow parameter. One warp and one weft systematic trend are enough to represent the mean tow path of a laminate. Similar conclusions are drawn for the deviations from the systematic values. The statistics of the fluctuations for different plies are indistinguishable, permitting the derivation of the standard deviation and correlation length of each parameter using the data set of deviations combining data from all plies. This increase of data set size is favoured, certainly for the correlation information which is sensitive to the amount of data.

Statistical information (σ, ξ_{auto}) quantified in [9] is given in tables 1 and 2. Correlation lengths are derived using linear regression of the correlation data corresponding to only the first five point spacings (k=5) due to the small data set size. The deviations are fairly represented by a normal distribution with the in-plane centroid subjected to the largest variability. This reflects the difference in tow tension for warp and weft tows during production. All tow characteristics, with exception of the in-plane centroid, vary within the unit cell dimensions. This is indicated by the correlation length which exceeds the unit cell size for the in-plane position as shown in table 2, while the out-of-plane centroid possesses a correlation length in the range of 2-4 mm. The in-plane component is less controlled than the out-of-plane centroid during RTM production where flat platens fix the thickness of the laminate. The area has a correlation length similar to the cross-over spacings, while the aspect ratio and orientation vary within the unit cell size. Generally, large uncertainty is present when deriving the correlation lengths using only data of the single plies. The limited size of the data set for the largest point separations causes fitted lines to be affected by outliers, which is a major source of uncertainty in the correlation length. Additional data must be collected to bring down this variation. Cross-correlations between tows of the same or different genuses are investigated in [11] for neighbouring locations. However, only weak dependencies are found thus no cross-correlation lengths are computed. Limited correlation (0.4 - 0.6) between the in-plane centroid of neighbouring tows is present, both for the warp and weft genus. The correlation of the warp and weft tow at the cross-over locations is found to be weak for the z-centroid, which is expected due to the RTM production process that fixes the thickness of the composite. For further details and discussion of the results the reader is referred to [9] and [11].

3.2.3. Long-range characterisation

The short-range characterisation of the tow path concludes that only the in-plane component varies at the long-range (more than one periodic length). Additional data of this centroid is collected in [10] by optical imaging of two one-ply samples spanning a region of thirteen unit cells by thirteen unit cells. The production of one-ply samples is more challenging than multiple-ply samples but offers the advantage to obtain a high contrast between tow and resin regions for the image processing step. The in-plane dimension of both samples (sample 1 & 2, $s = 1..2$) is scanned with a resolution of 1200 dots per inch (DPI), with figure 5 showing the digital image of sample 1. In-plane centrelines of forty warp and forty weft tows ($N_j = 40$) are quantified within the region of interest by visual recognition of the tow boundaries using the freeware GIMP. Centroid locations are subsequently defined as half the tow width at each grid location. These coordinates are given as input to Matlab where they are transformed to a global coordinate system and compensated for possible misalignment during scanning of the sample.

In-plane undulations, as shown in Figure 5, are quantified by the difference between the experimental tow paths and an ideal lattice description. Tows of this lattice are represented as straight lines with nominal spacing in x- and y-direction derived from the experimentally obtained periodic lengths λ_x and λ_y. In total, forty data points are extracted ($N_i = 40$) with the in-plane warp and weft deviations considered in respectively the y- and x-direction.

The obtained differences are further decomposed in a mean trend and zero-mean deviations using equation 1 with s=1 or 2 referring to the sample. However, the particular average trend should not be interpreted as a systematic periodic trend but as a handling trend. Due to the handling of the material during storage, cutting and placement in the RTM mould, shearing of the fabric occurs that affects the in-plane movement of the tows. This average pattern for the warp and weft genus is shown in figure 6. The stochastic variations of sample 1 and 2 are combined in one larger data set per tow genus, since no expected statistical differences are present between the samples.

The random behaviour of the in-plane deviations, approximately represented by a normal distribution, is further described in terms of standard deviation and correlation length along the tow and between tows (tables 1 and 2). The standard deviation of in the in-plane centroid is found to be five times higher for the weft direction. This is again attributed to the production process where warp tows are tensioned while weft tows are inserted. Correlation lengths are estimated by fitting appropriate correlation functions to the data in a least-square sense. The auto- and cross-correlation behaviour is well represented by an exponential correlation function $C_{exp} = \exp\left(-\frac{\tau}{\xi}\right)$ for the warp in-plane deviations, while the squared exponential function $C_{sq,exp} = \exp\left(-\frac{\tau^2}{\xi^2}\right)$ is a better fit for both correlation directions of the weft tows. The in-plane warp correlation length ξ_{auto} along the tow is approximately twice of the weft tows, reflecting the straightness of the warp tows and the significant in-plane movement of the weft tows. Distortions in the weft in-plane centroid positions affect near- and further-neighbouring tows, appearing as bands in the composite tow paths as shown in figure 5. This translates in a significantly high cross-correlation length ξ_{cross} for the weft tows, exceeding the unit cell dimension, while the cross-correlation of neighbouring warp tows are limited within the unit cell. A thorough discussion of the procedure and results is given in [10].

3.2.4. Summary of the statistical information of the carbon-epoxy 2/2 twill woven composite

Figure 6 shows the warp and weft systematic periodic trends of the out-of-plane centroid coordinate z, aspect ratio AR and area A, together with the handling effect of the in-plane centroid position. The periodic trend is defined over the unit cell dimension, while the handling effect is typified for a length of ten unit cells.

The short- and long-range statistical information is described in tables 1 and 2. Except for the in-plane centroid component, the short-range statistical information is considered to calibrate the stochastic modelling step.

4. Stochastic multi-scale modelling of the reinforcement (step 2)

4.1. Modelling strategy

Virtual specimens are created that possess the same statistical information on average. The statistical analysis concluded that there are two physical scales present in a textile: (1) the scale of the unit cell statistics, mainly determined by the cross-over locations at the meso-scale, and (2) the sub-component- or macro-scale, which reflects variations in the tow tensioning or insertion during manufacturing. To reproduce instances that include short- and long-range variations of all tow path parameters, a fairly large degree of freedom (DOF) system needs to be solved since a significantly small grid spacing is required to accurately represent the short-range variations of tow path parameters that vary on the meso-scale, identified by ϵ^{sr}, while the long-range deviations on the macro-scale, indicated by ϵ^{lr}, necessitates a large length of the grid to represent the long wavelengths. The generation of these zero-mean fluctuations results in a large computational model with simulated deviations $\tilde{\epsilon}^{sr}$ and $\tilde{\epsilon}^{lr}$ that need to be combined carefully. While micro-CT provides complete 3-D descriptions of the tow path within a unit cell, an optical scan results in rather incomplete 2-D surface information measured at typically one (or a few) point(s) per unit cell. A necessary condition for superposition of the short-range deviations onto the long-range deviations is that the short-and long-range deviations are not correlated with each other.

The simulation approach for woven composites consists of constructing a 2-D lattice of straight tows as pre-processing step, with rows representing warp tows and columns representing weft tows. On each row or column of the lattice, a stochastic realisation of the tow path is obtained by successive execution of the following steps:

1. Define an equidistant grid consisting of locations i'[1] and with a length $N_{i'}$ that (i) at least approximates the largest correlation length for each tow genus, (ii) corresponds to a multiple of the periodic length and (iii) has a spacing that is less than the shortest correlation length.
2. Interpolate the average tow path trends of all tow path parameters to match the grid locations. Periodicity is exploited to construct the repetitive systematic trend of the short-range parameters along the grid.
3. Generate a set of zero-mean deviations $\{\tilde{\epsilon}^{sr}_{i'}, i' = 1..N_{i'}\}$ along the grid for each tow path property that exhibits a short-range trend. This step is performed continuously along an entire tow path spanning multiple unit cells and does not need to be limited per unit cell length.
4. Generate a set of zero-mean deviations $\{\tilde{\epsilon}^{lr}_{i'}, i' = 1..N_{i'}\}$ along the grid for each tow path property that exhibits a long-range trend[2].

This procedure of generating realistic tow paths remains the same for any periodic type of textile. Only the pre-processing step in defining the lattice will be different according to the reinforcement topology.

The considered 2/2 twill composite has three DOFs at the meso or unit cell scale with ϵ^{sr}_{i} referring to the out-of-plane centroid z, cross-sectional area A or aspect ratio AR. The macro-scale is characterised by the in-plane centroid DOF ρ that corresponds to y or x, respectively for warp and weft genus. That more DOFs are needed to characterise deviations at the meso-scale than at the macro-scale was also found to be case for a 3-D interlock composite [13].

Tow path deviations of any property that are uncorrelated with neighbouring tows or other properties on the same tow (no type of cross-correlation), are generated with the Monte

[1] Grid spacing of i' does not coincide with the grid spacing of i for the short-range data.

[2] An alternative approach is to define a second grid of the same length but with a considerably larger grid spacing, with at least one point per periodic length, and afterwards interpolate the produced values to the short-range grid. This is valid if the correlation length of the long-range parameter is considerably larger than the unit cell size.

Carlo Markov Chain method for textile structures [15]. All deviations for the particular parameter of one tow are produced in a single process. The Markovian procedure is the core computation within the Monte Carlo based scheme and generates the distribution vector P^{ϵ}_{i+1} of the particular parameter ϵ at the next grid location $i+1$ using a probability transition matrix A^{ϵ}_{trans}:

$$P^{\epsilon}_{i+1} = A^{\epsilon}_{trans} P^{\epsilon}_{i} \tag{2}$$

This probability transition matrix is unique for each tow parameter and is calibrated with the experimental standard deviation and nearest neighbour (k=1) correlation information. An additional smoothing operation reduces the numerical noise on top of the low-amplitude short-range wavelength variations, which is associated with the discreteness of the Markov Chain. More details on this procedure can be found in [15, 11].

Tow path fluctuations that are correlated along the tow and between neighbouring tows require a simultaneous generation of all deviations belonging to the same genus within the specimen. This cross-correlated behaviour in textiles is expected for the centroid positions. The methodology of Vořechovský [16] based on Series Expansion can generate characteristics that share the same auto-correlation and of which the cross-correlation can be represented by a cross-correlation coefficient. The algorithm is calibrated with the experimental standard deviation, auto-correlation and cross-correlation statistics, of which the correlation information is summarised in correlation matrices. Tow path characteristics can be represented by a random matrix $\mathbf{H}$: $\mathbf{H} = \{[H^1]^T [H^2]^T ... [H^{N_j}]^T\}$ with $H^j = [\epsilon^j_1 \epsilon^j_2 ... \epsilon^j_{N_i}]$. Each row in $\mathbf{H}$ corresponds to the tow path deviations of a single tow which are defined at equidistant positions along its length. A single realisation of the deviations of each tow path H_j, represented by $\tilde{H}_j$, is acquired using the Karhunen-Loève (K-L) Series Expansion [17], with λ and ϕ respectively the eigenvalues and -vectors of the auto-correlation matrix:

$$\tilde{H}_j(x) = \sum_{i=1}^{N_{var}} \sqrt{\lambda^A_i} \chi^D_{j,i} \phi^A_i(x) \tag{3}$$

The random variables within χ^D are uncorrelated for each set of random variables that describe a single tow and at the same time cross-correlated between all sets of random variables belonging to different tows. A detailed discussion of the procedure and truncation possibility N_{var} of the K-L terms, applied to textile composites, is elaborated in [12]. An alternative but similar approach to represent cross-correlated positional deviations would consist in representing the positional deviations by a discrete Fourier Transform where the amplitudes are generated by a Markov Chain, while the phases of the Fourier components are produced via a random walk algorithm; details on this method may be found in [18].

4.2. Application to a 2/2 twill woven textile composite

Virtual models are generated for the 2/2 twill woven textile spanning a region of ten by ten unit cells. The model is representative for one ply within a laminate. Each tow is discretised in 320 equidistant points such that the information of one unit cell is defined over a grid of thirty-two points. The main assumptions in the modelling strategy are that (i) deviations are assumed to be normally distributed, (ii) the cross-section of a tow is approximated by an ellipse shape and (ii) short- and long-range deviations can be generated independently from each other.

Zero-mean fluctuations for the out-of-plane centroid positions z, tow aspect ratio AR and tow area A are produced with the Monte Carlo Markov Chain algorithm. Each tow parameter is produced over the grid of 320 points. Based on the statistical information of the tow path parameters, it is sufficient to discretise the experimental deviations in twenty pieces with corresponding distribution vector. Smoothing is applied as post-processing step using information of ± 2 neighbouring grid points. This simulation approach reproduces the wavelengths of fluctuations observed in the experiments.

The cross-correlated in-plane centroid positions of the considered 2/2 twill woven composite are simulated by the cross-correlated K-L Series Expansion technique. The experimental correlation matrices are constructed by projecting the auto- and cross-correlation functions for each tow genus onto a grid of forty-one points that span at least ten times the periodic length of the tow direction (auto-correlation) or of the perpendicular tow direction (cross-correlation). Next, these produced deviations are interpolated over the same grid of 320 points on which the short-range variations are defined. The short wavelength of the warp deviations and long wavelength of the weft variations are reproduced by the simulations, without the need of an additional smoothing operation.

5. Construction of virtual specimens in a geometrical modelling software (step 3)

This step is illustrated by using the commercial WiseTex software [19]. The reinforcement description of a nominal WiseTex model with the same topology is overwritten with the stochastic tow path instantiations, acquired as combination of average trends with the produced zero-mean deviations from step 2.

An arbitrary virtual specimen of the 2/2 twill woven composite is presented in figure 7, with the warp and weft tows oriented respectively along the horizontal and vertical axis. A significant difference in the in-plane centroid mobility of warp and weft tows is observed: weft tows are subjected to more in-plane movement with at some locations clustering of adjacent tows. The unit cell image demonstrates the variation in the out-of-plane centroid position and tow cross-sectional parameters.

6. Discussion

It is important to assess the errors that are introduced when (i) quantifying the internal structure of a composite by experiments and (ii) performing the statistical analysis. First, the characterisation of a geometrical parameter is carried out by image processing where inaccuracies are introduced in e.g. the image segmentation and which geometrical shape is considered to represent a tow's cross-section. In the statistical analysis, a distribution type is proposed which approximates the real physical probability density function and its corresponding distribution parameters. Further, a large uncertainty is present for the estimation of the correlation length. A representative correlation function can only be derived for sufficiently large data sets for each of the point distances in the correlation graph.

These virtual specimens can be further employed to (i) predict its mechanical performance, such as stiffness, in order to have a quantitative measure of the spatial variation over the structure, (ii) analyse more precisely the damage progression in textile composites, or (iii) accurately simulate the resin impregnation of component-size fabrics. The WiseTex format is directly compatible with tools for micromechanical analysis [20] and permeability simulations [21]. However, when transforming the WiseTex model into a finite element model, small adjustments are required in the tow path description since limited interpenetration appears in the virtual specimens, as observed in figure 7.

7. Conclusions

This paper is a contribution towards an improved comprehension of the material performance by identifying the inherent uncertainty in the reinforcement geometry. The presented experimental methodologies and probabilistic uncertainty approach provide a solution to the lack of mature experimental characterisation schemes with a complete statistical analysis to efficiently and accurately determine the geometrical randomness. A roadmap is defined to build a complete statistical description of the random reinforcement at the short-range (meso-scale) and long-range (macro-scale) of any textile composite. Average trend, standard deviation as well as correlation information of each tow path parameter are essential information to feed advanced simulation techniques that can construct realistic textile models. Stochastic simulation based on experimental data can relate the variation in the mechanical properties with the geometrical characteristics at the lower scale. The methodology is demonstrated for a carbon-epoxy 2/2 twill woven composite produced by RTM, but all statistical methods and simulation tools are generically described to model any textile topology considering only minor modifications.

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Figures

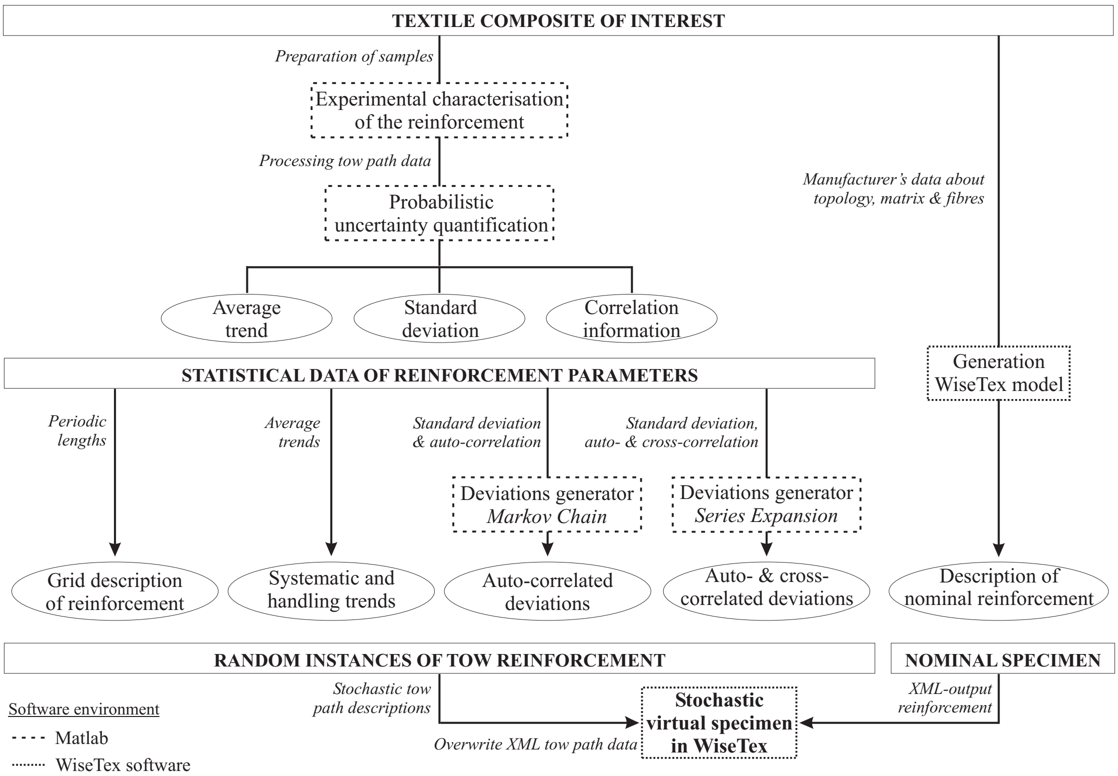

Figure 1: Multi-scale framework.

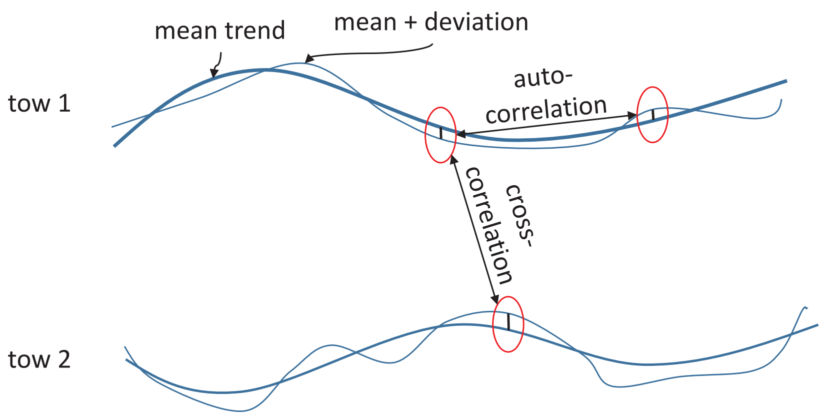

Figure 2: Definition of spatial dependencies of tow path deviations: auto-correlation (along the tow) and cross-correlation (between tows).

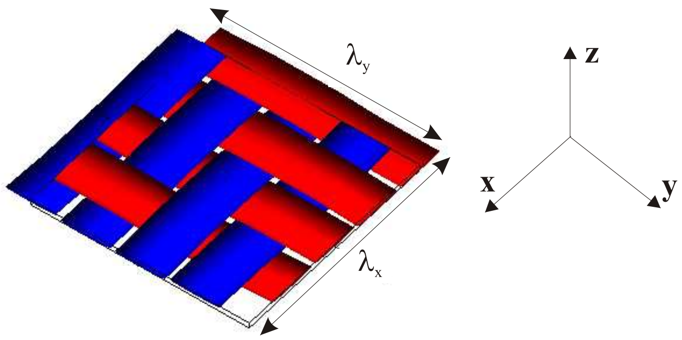

Figure 3: WiseTex model of a 2/2 twill woven reinforcement. The x-axis and y-axis of the coordinate system are respectively parallel to the warp and weft direction.

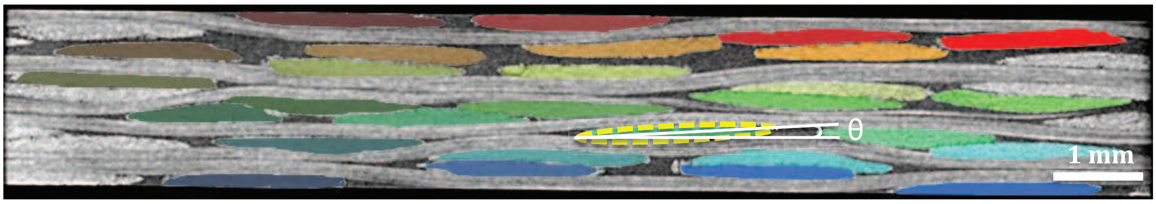

Figure 4: Digital image of a cross-section in weft direction obtained from micro-CT. Tows in cross-section are segmented and afterwards fitted with an ellipse shape.

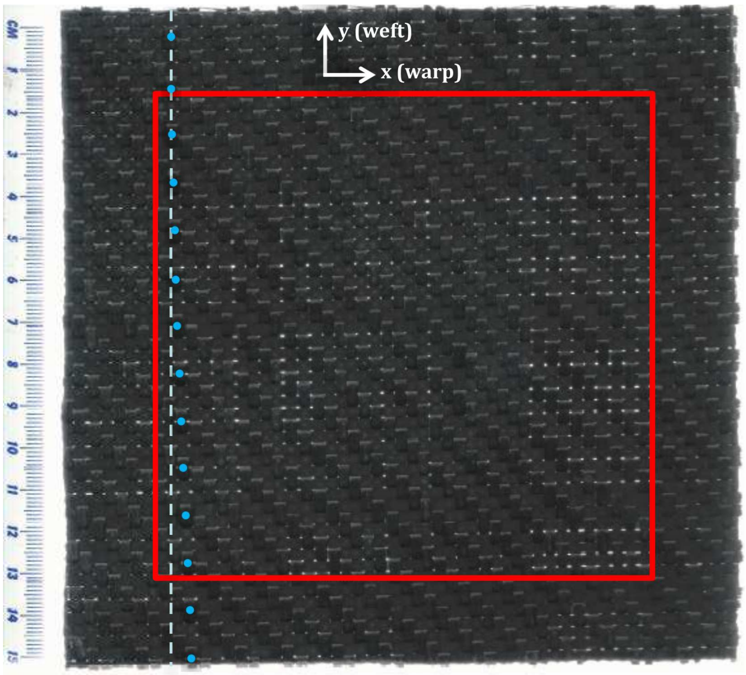

Figure 5: Optical scan of a one-ply 2/2 twill woven carbon fibre fabric impregnated with epoxy resin. The red square indicates the region of interest.

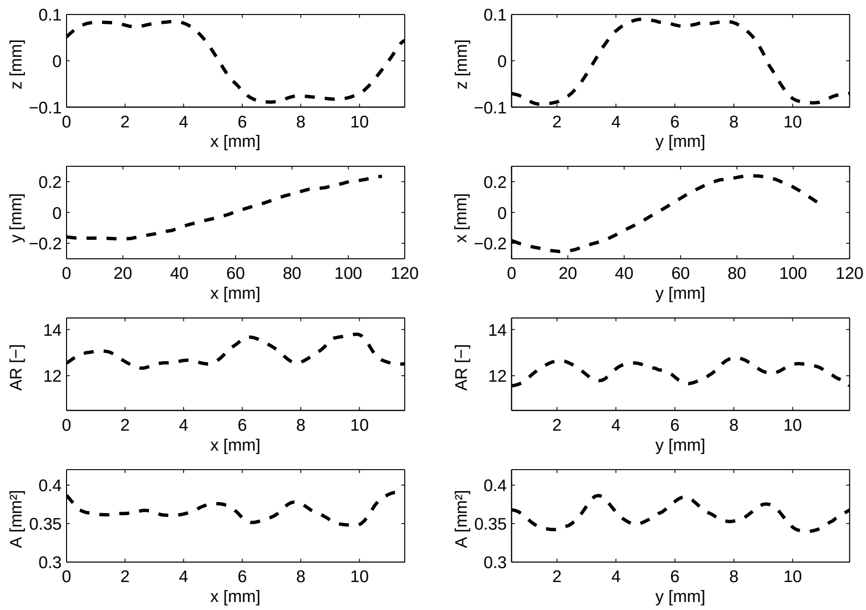

Figure 6: Periodic and handling trends of a 2/2 twill woven carbon-epoxy composite for the warp (along x-axis) and weft genuses (along y-axis).

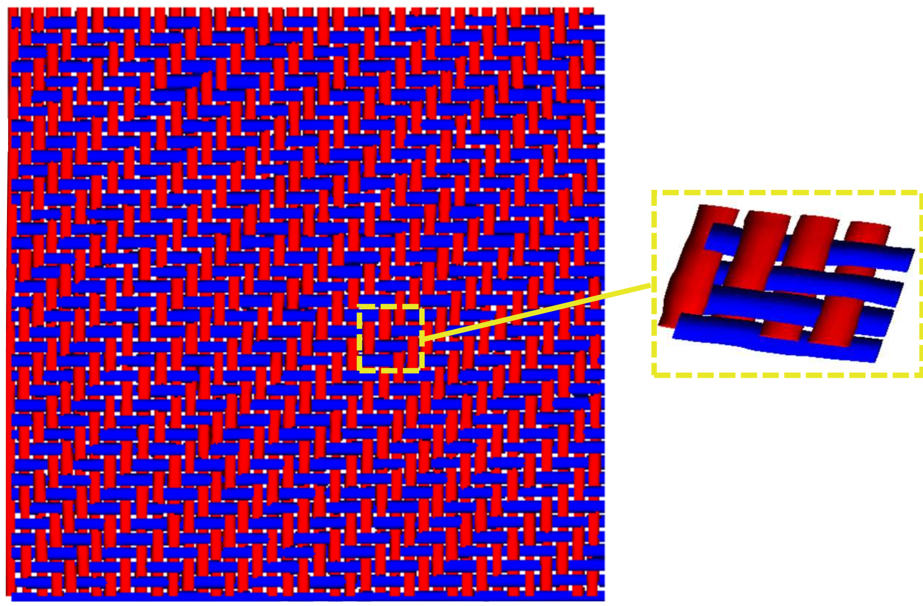

Figure 7: WiseTex representation of a virtual specimen spanning multiple unit cells.

Tables

Table 1: Standard deviation of the tow path parameters from the short-range [9] and long-range characterisation [10], respectively indicated by *sr* and *lr*.

	σ_x^{sr} [mm]	σ_y^{sr} [mm]	σ_z^{sr} [mm]	σ_{AR}^{sr} [-]	σ_A^{sr} [mm^2]	σ_θ^{sr} [°]	σ_x^{lr} [mm]	σ_y^{lr} [mm]
σ^{warp}	-	0.113	0.014	1.774	0.023	0.797	-	0.106
σ^{weft}	0.063	-	0.015	1.440	0.024	0.833	0.615	-

Table 2: Correlation lengths of the tow path parameters from the short-range [9] and long-range characterisation [10], respectively indicated by *sr* and *lr*. Only for the in-plane position a cross-correlation length is defined.

	ξ_x^{sr} [mm]	ξ_y^{sr} [mm]	ξ_z^{sr} [mm]	ξ_{AR}^{sr} [mm]	ξ_A^{sr} [mm]	ξ_θ^{sr} [mm]	ξ_x^{lr} [mm]	ξ_y^{lr} [mm]
ξ_{auto}^{warp}	-	22.89	1.78	7.26	2.53	4.56	-	114.89
ξ_{cross}^{warp}	-	-	-	-	-	-	-	4.49
ξ_{auto}^{weft}	9.42	-	1.62	5.48	1.01	3.49	52.89	-
ξ_{cross}^{weft}	-	-	-	-	-	-	13.16	-